

Solving the Inverse Electromagnetic Problem of Reconstructing Inhomogeneity in a Dielectric Body by Near-Field Measurements using the Two-Step Method

Yu. G. Smirnov^a and A. O. Lapich^{a,*}

^a Penza State University, Penza, 440026 Russia

*e-mail: lapich.a@yandex.ru

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Abstract—Background. The inverse electromagnetic problems of inhomogeneity reconstruction in a dielectric body by the near-field measurements arise, for example, in the early diagnosis of breast cancer by microwave tomography. The solution of such inverse problems is the basis for the development of a technique for detecting inhomogeneities using microwave devices. The efficiency of the technique depends on the inverse problem solution accuracy. Therefore, the development of new, more accurate methods for solving the inverse problem of microwave tomography is highly relevant. **Aim.** This study is aimed at the development of a method for solving the electromagnetic inverse problem of microwave tomography using near-field measurements, i.e., restoring the structure of an inhomogeneous dielectric body from values of an electromagnetic field outside this body using a measuring setup. **Methods.** The inverse problem is solved by a two-step method for determining the inhomogeneity of a body, in which the current function inside the body is first found and then the permittivity function is calculated. The method is non-iterative and does not require a good initial approximation. **Results.** A two-step method has been used to solve the inverse problem of microwave tomography. The numerical results are presented. Inhomogeneous bodies in the shape of a hemisphere have been investigated. Experimental results are presented. **Conclusions.** The efficiency of the proposed technique for detecting inhomogeneities in a dielectric body by microwave tomography is demonstrated. The calculation and experimental data are reported.

Keywords: electrodynamics problem, Maxwell's system of equations, integral equation, numerical method, microwave tomography, vector network analyzer

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INTRODUCTION

Inverse electromagnetic (vector) problems on restoring the structure of an inhomogeneous body have attracted considerable interest for several decades. One of the most popular approaches to solving these problems is the minimization of certain error functionals (using the Tikhonov regularization) and the use of iterative methods, which require a good initial approximation.

In this work, we use a non-iterative method to solve the inverse electromagnetic problem on restoring the structure of an inhomogeneous body impinged by a monochromatic electromagnetic wave. The problem consists in finding the unknown permittivity (or the corresponding refractive index) of a bounded bulk scatterer located in space on the basis of near-field measurements outside the body. A description, justification, and application of the two-step method are presented.

A direct problem of the diffraction of a monochromatic electromagnetic wave by a bounded bulk scat-

terer with a specified constant permeability and a known permittivity is formulated. The initial boundary value problem for Maxwell's equations is reduced to the system consisting of a singular integro-differential equation for the electric field over the inhomogeneity region and the integral representation of the total electric field outside the scatterer. The main results on the solvability of the direct diffraction problem are presented.

The inverse problem on finding the unknown permittivity of a volumetric body of a specified shape is then solved. It is shown that the integro-differential equation of the first kind has no more than one solution in finite-dimensional spaces of piecewise constant functions.

1. FORMULATION OF THE PROBLEM

Let there be some body $Q \subset \mathbb{R}^3$, which is a hemisphere; ∂Q is the piecewise smooth boundary. We

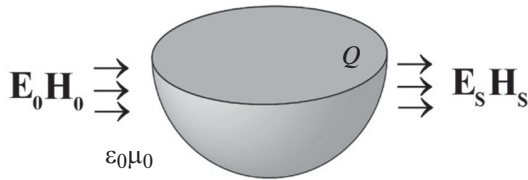


Fig. 1. Graphic illustration of the problem.

assume that the dielectric body Q is isotropic and inhomogeneous.

A medium in the space outside the body is assumed to be homogeneous with constant permeability μ_0 and permittivity ε_0 .

The field is excited by a point radiation source at the point $x_0 \in \mathbb{R}^3 \setminus \bar{Q}$ exciting the electromagnetic wave $\mathbf{E}_0, \mathbf{H}_0$, which satisfies Maxwell's system of equations outside this point:

$$\begin{cases} \text{rot } \mathbf{H}_0 = -i\omega\varepsilon_0\mathbf{E}_0, \\ \text{rot } \mathbf{E}_0 = i\omega\mu_0\mathbf{H}_0. \end{cases} \quad (1)$$

The total electromagnetic field at a point can be presented by the sum of two components: the incident field $\mathbf{E}_0, \mathbf{H}_0$ and the field $\mathbf{E}_s, \mathbf{H}_s$ scattered by an object (Fig. 1):

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_s, \quad \mathbf{H} = \mathbf{H}_0 + \mathbf{H}_s. \quad (2)$$

The solution of the direct diffraction problem, i.e., the total electromagnetic field $\mathbf{E}, \mathbf{H}$, satisfies in $\mathbb{R}^3 \setminus \partial Q$ Maxwell's equations

$$\begin{cases} \text{rot } \mathbf{H} = -i\omega\varepsilon\mathbf{E}, \\ \text{rot } \mathbf{E} = i\omega\mu_0\mathbf{H}. \end{cases} \quad (3)$$

We assume that, at the interface between two media, the conditions of continuity of the tangential field components at the boundary of the inhomogeneity region are satisfied:

$$[\mathbf{E}_\tau]_{\partial Q} = [\mathbf{H}_\tau]_{\partial Q} = 0, \quad (4)$$

and the conditions of finite energy in any limited volume of space are met:

$$\mathbf{E}, \mathbf{H} \in L_{2,\text{loc}}(\mathbb{R}^3). \quad (5)$$

A detailed formulation of problem (1)–(5) and study of its solvability are available in [1].

Boundary value problem (1)–(5) can be reduced [1] to a system consisting of the integro-differential equation over the inhomogeneity region

$$\begin{aligned} & \mathbf{E}(x) - (k_0^2 + \text{grad div}) \\ & \times \int_Q G(x, y)(\varepsilon_r(y) - 1)\mathbf{E}(y)dy = \mathbf{E}_0(x), \end{aligned} \quad (6)$$

$$x \in Q,$$

and the integral representation of the field outside the body,

$$\begin{aligned} \mathbf{E}(x) &= \mathbf{E}_0(x) + (k_0^2 + \text{grad div}) \\ & \times \int_Q G(x, y)(\varepsilon_r(y) - 1)\mathbf{E}(y)dy, \end{aligned} \quad (7)$$

$$x \in \mathbb{R}^3 \setminus \bar{Q},$$

where $G(x, y) = \frac{e^{ik_0|x-y|}}{4\pi|x-y|}$ and $\varepsilon_r = \varepsilon/\varepsilon_0$ is the relative permittivity.

The magnetic field is everywhere expressed through the electric field by the formula

$$\mathbf{H} = \frac{1}{i\omega\mu_0} \text{rot } \mathbf{E}.$$

In the region Q , we introduce the vector function

$$\mathbf{J}(x) := (\varepsilon_r(x) - 1)\mathbf{E}(x),$$

assuming that, everywhere in Q , the following condition is met: $|\varepsilon_r(x)| \geq \tilde{\varepsilon} > 1$. Then, from the representation of the field outside the scatterer, we obtain the equation for $\mathbf{J}(x)$

$$\begin{aligned} (k_0^2 + \text{grad div}) \int_Q G(x, y)\mathbf{J}(y)dy &= \mathbf{E}(x) - \mathbf{E}_0(x), \\ x &\in D, \end{aligned} \quad (8)$$

and rewrite the equation in the inhomogeneity region in the form

$$\begin{aligned} \frac{\mathbf{J}(x)}{\varepsilon_r(x) - 1} - (k_0^2 + \text{grad div}) \int_Q G(x, y)\mathbf{J}(y)dy &= \mathbf{E}_0(x), \\ x &\in Q. \end{aligned} \quad (9)$$

To solve the inverse problem on finding the unknown permittivity (or the corresponding refractive index) of a limited bulk scatterer located in space from the results of near-field measurements outside the body, we apply a two-step method [2–3]. The first step of the method consists in solving a linear integral equation of the first kind with respect to the polarization current (using the known values of the incident field $\mathbf{E}_0(x)$ and the total field $\mathbf{E}(x)$ in certain region D , it is necessary to find the current $\mathbf{J}(x)$ outside the body in Q from Eq. (8)). At the second step, $\varepsilon(x)$ is explicitly expressed through the known $\mathbf{J}(x)$ function using Eq. (9).

The numerical implementation of the two-step method was described in detail in [4–7]. The results of the calculation for a hemispherical body by this method are presented below.

Figure 2 illustrates the solution of the direct (on the left) and inverse (on the right) problems for a hemisphere containing nonuniformly distributed inhomogeneities with different refractive indices, the values of which are shown on a color scale. The inhomogene-

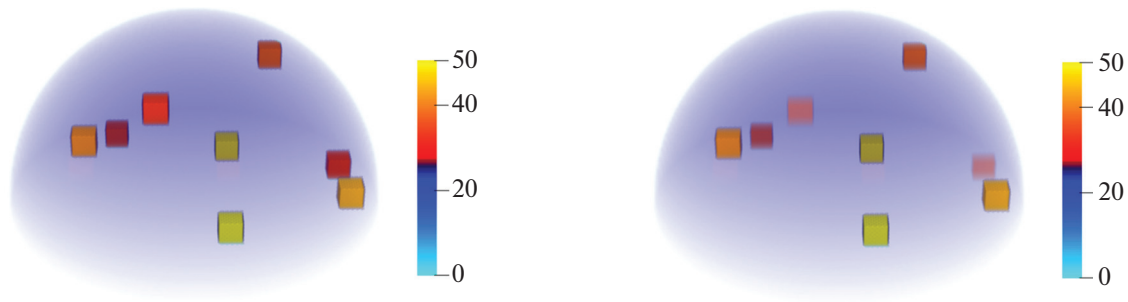


Fig. 2. Solving the direct and inverse problem for a hemispherical body.

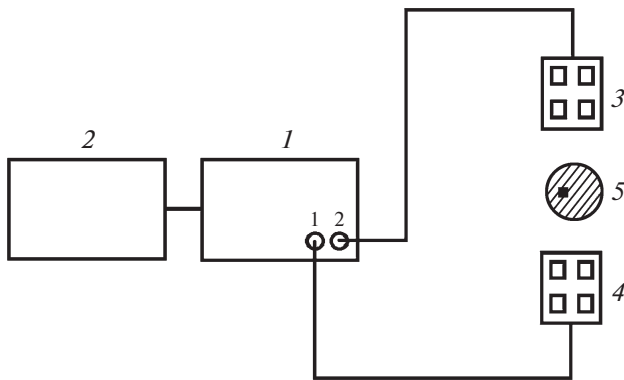


Fig. 3. Schematic of the measuring setup.

ities are visualized as colored figures within the hemisphere. It can be seen in Fig. 2 that the inhomogeneity values in the direct (exact values) and inverse (calculated approximate values) problems change insignificantly. The reconstructed data on their positions and inhomogeneity parameters are clearly distinguishable.

Figure 3 presents a schematic of the measuring setup implemented using a Rohde & Schwarz ZNLE6

two-port vector network analyzer (1 in Fig. 3) and a personal computer for analyzing and processing the obtained measurement data (2 in Fig. 3). The measurements were carried out in the frequency range from 5 to 5.5 GHz using microstrip printed antennas directed towards each other (3 and 4 in Fig. 3). During the measurements, the investigated sample with some inhomogeneity inside (5 in Fig. 3) is placed between two antennas. The investigated objects are hemispheres with inhomogeneities of different sizes inside: cubic inhomogeneities with sides of 1, 2, and 3 cm and a reference homogeneous object. Below, these options are designated by F_1, F_2, F_3, F_0 , respectively

Figures 4 and 5 show the absolute values of the real part of the measured reflectance and transmittance for objects with and without inhomogeneities. It can be seen from the plots that the parameter values change significantly with the inhomogeneity size. The experiments were also conducted at different positions of inhomogeneities (sample rotation) and, in these cases, showed changes in the S parameters as well.

The experimental data demonstrate the possibility of detecting inhomogeneities in a dielectric body using the electromagnetic field measurements in the near zone.

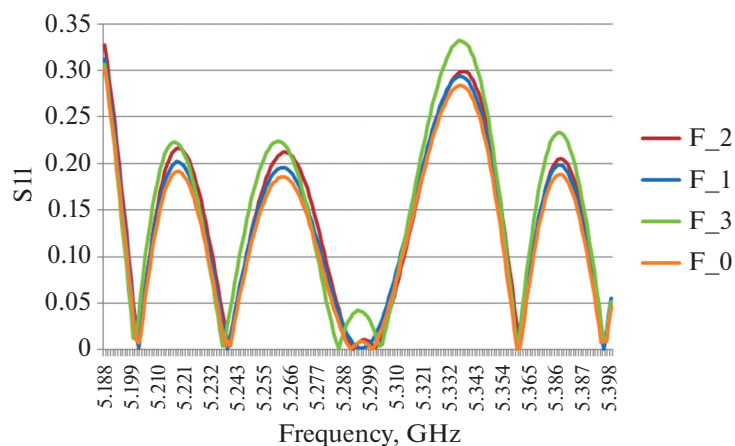


Fig. 4. Values of the S11 parameter for objects F_0, F_1, F_2, F_3 .

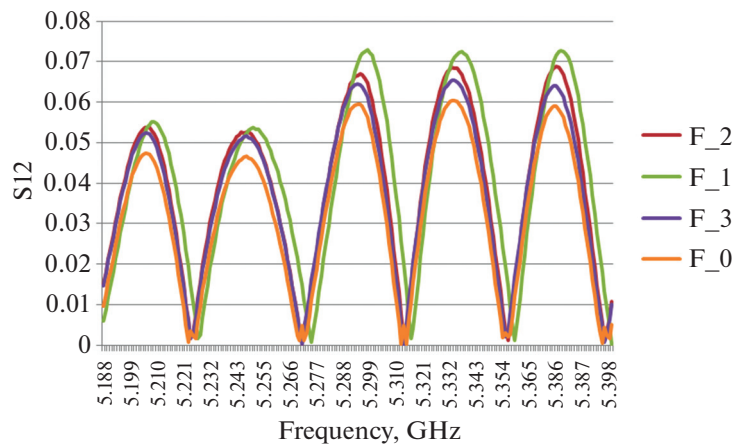


Fig. 5. Values of the S12 parameter for objects F_0, F_1, F_2, F_3 .

CONCLUSIONS

The method was discussed for solving the electromagnetic inverse problem of microwave tomography using near-field measurements, specifically, restoring the structure of an inhomogeneous dielectric body from the electromagnetic field values outside the body using a measuring setup. The inverse problem was solved by a two-step non-iterative method.

The presented numerical results for a hemispherical body showed the possibility of restoring fairly accurately the structure of an inhomogeneous body by measuring the field outside it.

The experimental data obtained using the measuring setup demonstrated the possibility of detecting relatively small, arbitrarily positioned inhomogeneities in a dielectric body by measuring the electromagnetic field in the near zone at different frequencies.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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