

PARTIAL DIFFERENTIAL EQUATIONS

On the Existence and Uniqueness of the Solution of an Integro-Differential Equation in the Problem of Diffraction of an Electromagnetic Wave by a Graphene-Coated Inhomogeneous Dielectric Body

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Received March 10, 2025; revised March 10, 2025; accepted June 20, 2025

Abstract—Boundary value problems for the system of Maxwell’s equations are fundamental in electrodynamics. Recently, interest has arisen in problems with the presence of a thin graphene layer on the surface, which changes the field-matching conditions. In this paper, an integro-differential equation is obtained for a vector boundary value problem of diffraction of an electromagnetic wave by a graphene-coated inhomogeneous dielectric body. The existence and uniqueness of a solution of the integro-differential equation, which can be called a surface-volume equation, is proved.

Keywords: integro-differential equation, Maxwell’s system of equations, dielectric body, graphene

DOI: 10.1134/S09655425252701003

INTRODUCTION

Boundary value problems for Maxwell’s equations with field matching play an important role in electrodynamics, especially in problems of electromagnetic-wave diffraction by bounded dielectric bodies.

Although field-matching problems have been studied quite well (see [1–5]), boundary-value problems with special field matching conditions that allow the presence of a thin layer of material (e.g., graphene) at the interface between the media have now begun to be studied (see [6–13]). In some cases, it is necessary to take into account the nonlinearity of the field matching conditions (due to the nonlinearity of graphene in the infrared and terahertz frequency ranges).

In [14], the problem of diffraction of an electromagnetic wave by a graphene-coated homogeneous dielectric body was considered. It was reduced to a system of surface integro-differential equations. In the present article, due to the inhomogeneity of the dielectric body, this approach is not applicable.

To study the problem of electromagnetic-wave diffraction by a graphene-coated inhomogeneous dielectric body, the method of volume singular integral equations was used, which reduces the boundary value field-matching problem to a volume integro-differential equation over the body. This approach also allows using numerical methods to find a solution to the integro-differential equation and facilitates efficient parallelization for solving on supercomputers.

The article considers a vector problem of diffraction of electromagnetic waves by an inhomogeneous dielectric body with a graphene coating (without taking into account the nonlinearity of graphene) and obtains a system of integro-differential equations over the inhomogeneous body. Continuous invertibility of the operator corresponding to the equation is proved, as well as the existence and uniqueness of a solution to the integro-differential equation.

1. PROBLEM STATEMENT

Consider a bounded domain $\Omega_1 \in \mathbb{R}^3$ with a smooth boundary Γ of class C^2 ; the permittivity and permeability in it are, respectively, $\varepsilon(x)$ and μ_0 and, in its exterior $\Omega_2 := \mathbb{R}^3 \setminus \bar{\Omega}_1$, ε_0 and μ_0 , where

$\varepsilon(x) \in C^\infty(\bar{\Omega}_1)$, $\operatorname{Re} \varepsilon(x) > 0$, and $\operatorname{Im} \varepsilon(x) \geq 0$ at $x \in \bar{\Omega}_1$. Thus, we are considering an inhomogeneous dielectric body.

The total electromagnetic field $(\mathbf{E}, \mathbf{H})$ in the domain Ω_1 is denoted as $(\mathbf{E}_1, \mathbf{H}_1)$, and in the domain Ω_2 , as $(\mathbf{E}_2, \mathbf{H}_2)$. The field is excited by an electromagnetic wave $(\mathbf{E}_0, \mathbf{H}_0)$, so that $\mathbf{E}_2 = \mathbf{E}_0 + \mathbf{E}_s$, $\mathbf{H}_2 = \mathbf{H}_0 + \mathbf{H}_s$, where $(\mathbf{E}_0, \mathbf{H}_0)$ is the known incident field and $(\mathbf{E}_s, \mathbf{H}_s)$ is the scattered field. The incident field satisfies the system of Maxwell's equations

$$\begin{cases} \operatorname{curl} \mathbf{H}_0 = -i\omega\varepsilon_0\mathbf{E}_0, \\ \operatorname{curl} \mathbf{E}_0 = i\omega\mu_0\mathbf{H}_0, \end{cases} \quad \text{in } \mathbf{R}^3. \quad (1)$$

To determine the indicated components, it is necessary to solve the system of Maxwell's equations

$$\begin{cases} \operatorname{curl} \mathbf{H}_1 = -i\omega\varepsilon(x)\mathbf{E}_1, \\ \operatorname{curl} \mathbf{E}_1 = i\omega\mu_0\mathbf{H}_1, \end{cases} \quad \text{in } \Omega_1, \quad (2)$$

$$\begin{cases} \operatorname{curl} \mathbf{H}_2 = -i\omega\varepsilon_0\mathbf{E}_2, \\ \operatorname{curl} \mathbf{E}_2 = i\omega\mu_0\mathbf{H}_2, \end{cases} \quad \text{in } \Omega_2, \quad (3)$$

where ω is the circular frequency.

At the boundary Γ , we impose the field matching conditions

$$[\mathbf{v} \times \mathbf{H}]_\Gamma = \sigma \mathbf{E}_\tau, \quad (4)$$

$$[\mathbf{v} \times \mathbf{E}]_\Gamma = 0, \quad (5)$$

where $[f]_\Gamma = f_2 - f_1$ denotes the difference in the traces of the function from different sides of Γ , $\mathbf{v}$ is the vector normal to the boundary Γ directed into the domain Ω_2 , the index τ denotes the tangent components, $(\times)$ denotes the vector product, and

$$\sigma = \sigma_A \sigma_g(x) \quad (6)$$

is the linear conductivity of graphene—a complex-valued function [15], where $\sigma_A > 0$ and $|\sigma_g(x)| \leq 1$; $\sigma_g(x) \in C^\infty(\Gamma)$.

At infinity, we have to satisfy the Silver–Muller conditions (see [4, 5]):

$$(\mathbf{e}_r \times \mathbf{E}_s) + \sqrt{\frac{\mu_0}{\varepsilon_0}} (\mathbf{e}_r \times (\mathbf{e}_r \times \mathbf{H}_s)) = o\left(\frac{1}{r}\right) \quad \text{as } r \rightarrow \infty, \quad (7)$$

where $\mathbf{e}_r$ is the normal vector to the unit sphere, $r := |x|$, and $x = (x_1, x_2, x_3)$.

We will assume that $\mathbf{E}, \mathbf{H} \in C^1(\Omega_1) \cap C(\bar{\Omega}_1)$ and $\mathbf{E}, \mathbf{H} \in C^1(\Omega_2) \cap C(\bar{\Omega}_2)$, i.e., seek the classical solution of problem (1)–(7).

2. INTEGRO-DIFFERENTIAL EQUATION

We define

$$\Phi(x, y) = \frac{e^{ik_0|x-y|}}{4\pi|x-y|}, \quad k_0^2 = \omega^2\varepsilon_0\mu_0.$$

Using the Stratton–Chu formulas, we express the fields $\mathbf{E}$ and $\mathbf{H}$ via a combination of surface electric and magnetic currents.

In the domain Ω_1 , by virtue of (1), we have [4], p. 124, Theorem 4.1,

$$-\mathbf{E}_0(x) = \operatorname{curl} \int_\Gamma \Phi(x, y) (\mathbf{v}_1(y) \times \mathbf{E}_0(y)) ds(y) - \frac{1}{i\omega\varepsilon_0} \operatorname{curl} \operatorname{curl} \int_\Gamma \Phi(x, y) (\mathbf{v}_1(y) \times \mathbf{H}_0(y)) ds(y), \quad (8)$$

where $x \in \Omega_1$ and $\mathbf{v}_1$ is the outer normal to the boundary of the domain Ω_1 . Hereinafter, we take into account that the notation of the systems of Maxwell's equations (1)–(3) differs in coefficients from the similar notation in [4], but it is easy to recalculate them, so we will not dwell on this issue.

Further, by virtue of (3) and (7), we also have [4], p. 128, Theorem 4.5

$$0 = \operatorname{curl} \int_{\Gamma} \Phi(x, y) (\mathbf{v}_2(y) \times \mathbf{E}_s(y)) ds(y) - \frac{1}{i\omega\epsilon_0} \operatorname{curl} \operatorname{curl} \int_{\Gamma} \Phi(x, y) (\mathbf{v}_2(y) \times \mathbf{H}_s(y)) ds(y), \quad (9)$$

where $x \in \Omega_1$ and $\mathbf{v}_2$ is the outer normal to the boundary of the domain Ω_2 .

Subtracting Eq. (9) from (8), given that $\mathbf{v}_2 = -\mathbf{v}_1$, we get

$$-\mathbf{E}_0(x) = \operatorname{curl} \int_{\Gamma} \Phi(x, y) (\mathbf{v}_1(y) \times \mathbf{E}_2(y)) ds(y) - \frac{1}{i\omega\epsilon_0} \operatorname{curl} \operatorname{curl} \int_{\Gamma} \Phi(x, y) (\mathbf{v}_1(y) \times \mathbf{H}_2(y)) ds(y), \quad (10)$$

where $x \in \Omega_1$.

System (2) can be rewritten as

$$\begin{cases} \operatorname{curl} \mathbf{H}_1 = -i\omega\epsilon_0 \mathbf{E}_1 + \mathbf{J}, \\ \operatorname{curl} \mathbf{E}_1 = i\omega\mu_0 \mathbf{H}_1, \end{cases} \quad \text{in } \Omega_1, \quad (11)$$

where

$$\mathbf{J} := -(\epsilon(x)/\epsilon_0 - 1)i\omega\epsilon_0 \mathbf{E}_1. \quad (12)$$

Applying formula (6.5) from [5], p. 191, Theorem 6.1, by virtue of (11) and (12), we find

$$\begin{aligned} \mathbf{E}(x) = & -\operatorname{curl} \int_{\Gamma} \Phi(x, y) (\mathbf{v}_1(y) \times \mathbf{E}_1(y)) ds(y) + \operatorname{grad} \int_{\Gamma} \Phi(x, y) (\mathbf{v}_1(y) \cdot \mathbf{E}_1(y)) ds(y) \\ & - i\omega\mu_0 \int_{\Gamma} \Phi(x, y) (\mathbf{v}_1(y) \times \mathbf{H}_1(y)) ds(y) - \frac{1}{i\omega\epsilon_0} \operatorname{grad} \int_{\Omega_1} \Phi(x, y) \operatorname{div} \mathbf{J}(y) dy + i\omega\mu_0 \int_{\Omega_1} \Phi(x, y) \mathbf{J}(y) dy, \end{aligned}$$

where $x \in \Omega_1$. Next, we transform the obtained expression using the formulas of vector analysis, Stokes' theorem, and (11), (12) to

$$\begin{aligned} \mathbf{E}(x) = & -\operatorname{curl} \int_{\Gamma} \Phi(x, y) (\mathbf{v}_1(y) \times \mathbf{E}_1(y)) ds(y) + \frac{1}{i\omega\epsilon_0} \operatorname{curl} \operatorname{curl} \int_{\Gamma} \Phi(x, y) (\mathbf{v}_1(y) \times \mathbf{H}_1(y)) ds(y) \\ & - \frac{1}{i\omega\epsilon_0} \operatorname{grad} \operatorname{div} \int_{\Omega_1} \Phi(x, y) \mathbf{J}(y) dy + i\omega\mu_0 \int_{\Omega_1} \Phi(x, y) \mathbf{J}(y) dy, \end{aligned} \quad (13)$$

where $x \in \Omega_1$.

Now, adding (10) and (13), given (4), (5), and (12), we get

$$\begin{aligned} \mathbf{E}(x) = & \mathbf{E}_0(x) - \frac{1}{i\omega\epsilon_0} \operatorname{curl} \operatorname{curl} \int_{\Gamma} \Phi(x, y) (\sigma(y) \mathbf{E}_\tau(y)) ds(y) \\ & + \operatorname{grad} \int_{\Omega_1} \Phi(x, y) \operatorname{div} \left(\left(\frac{\epsilon(y)}{\epsilon_0} - 1 \right) \mathbf{E}(y) \right) dy + k_0^2 \int_{\Omega_1} \Phi(x, y) \left(\left(\frac{\epsilon(y)}{\epsilon_0} - 1 \right) \mathbf{E}(y) \right) dy, \end{aligned}$$

or, after transformations, we obtain the equation

$$\begin{aligned} \mathbf{E}(x) = & \mathbf{E}_0(x) - \frac{1}{i\omega\epsilon_0} (\operatorname{grad} \operatorname{div} + k_0^2) \int_{\Gamma} \Phi(x, y) (\sigma(y) \mathbf{E}_\tau(y)) ds(y) \\ & + (\operatorname{grad} \operatorname{div} + k_0^2) \int_{\Omega_1} \Phi(x, y) \left(\left(\frac{\epsilon(y)}{\epsilon_0} - 1 \right) \mathbf{E}(y) \right) dy, \end{aligned} \quad (14)$$

where $x \in \Omega_1$.

Note that all the above transformations are justified, since we consider solutions from the classes of functions $\mathbf{E}, \mathbf{H} \in C^1(\Omega_1) \cap C(\bar{\Omega}_1)$ and $\mathbf{E}, \mathbf{H} \in C^1(\Omega_2) \cap C(\bar{\Omega}_2)$, for which the formulas used above were obtained in [4, 5].

Equation (14) was first formally obtained in [16]. In this paper, it is obtained by another method.

This equation is a hypersingular integral equation if the gradient and divergence operations are introduced under the integral sign. However, we will not regard it as hypersingular. It will be more convenient for us to treat it as an integro-differential or pseudo-differential equation.

A specific feature of this equation is that it contains both the unknown function $\mathbf{E}$ itself in the domain Ω_1 and its trace $\mathbf{E}_\tau$ on the boundary Γ . It can be called a surface-volume equation.

3. SOLVABILITY OF THE INTEGRO-DIFFERENTIAL EQUATION

It is more convenient to study the integro-differential equation (14) not in the class of smooth functions, but in suitable Sobolev spaces. These spaces are selected from the conditions of finiteness of energy in any bounded spatial domain [17, p. 38]. The issues of smoothing of solutions to the equation under consideration are not considered in this article.

Let us introduce the space of functions (see [17, p. 40], [5, p. 207])

$$H = H(\Omega_1) \equiv L_{\text{Curl}}^2(\Omega_1) := \{\mathbf{F} : \mathbf{F} \in L_2(\Omega_1), \text{curl } \mathbf{F} \in L_2(\Omega_1)\}$$

with scalar product and norm

$$(\mathbf{F}, \mathbf{V})_H = (\mathbf{F}, \mathbf{V})_{L_2(\Omega_1)} + (\text{curl } \mathbf{F}, \text{curl } \mathbf{V})_{L_2(\Omega_1)}, \quad \|\mathbf{F}\|_H^2 = \|\mathbf{F}\|_{L_2(\Omega_1)}^2 + \|\text{curl } \mathbf{F}\|_{L_2(\Omega_1)}^2.$$

We also introduce the space (see [17, p. 86], [5, p. 208])

$$W = W(\Gamma) \equiv H_{\text{div}}^{-1/2}(\Gamma) := \{\mathbf{u} : \mathbf{u} \in H^{-1/2}(\Gamma), \text{div } \mathbf{u} \in H^{-1/2}(\Gamma)\}.$$

Such spaces W were first proposed in [18], then in [17], and later began to be used all over the world to solve problems of diffraction on screens (in foreign publications, they are usually denoted as $H_{\text{div}}^{-1/2}(\Gamma)$). Here, $\mathbf{u}$ is a tangent vector field. More precisely, it is a section of the vector bundle over the manifold Γ . The Sobolev space $H^{-1/2}(\Gamma)$ is usually defined via the Fourier transform in local maps and the partition of unity [17, p. 84]. By div we mean the operation of surface divergence [17, p. 84]. The scalar product and the norm in W are also defined in [17, p. 85].

Let us write Eq. (14) in operator form

$$\mathbf{E} = \mathbf{E}_0 - \sigma_A \frac{1}{i\omega\epsilon_0} L(\sigma_g T\mathbf{E}) + A\mathbf{E}, \quad (15)$$

where

$$L\mathbf{u} := \left(\text{grad div} + k_0^2 \right) \int_{\Gamma} \Phi(x, y) \mathbf{u}(y) ds(y), \quad (16)$$

$$A\mathbf{F} := \left(\text{grad div} + k_0^2 \right) \int_{\Omega_1} \Phi(x, y) (\epsilon(x)/\epsilon_0 - 1) \mathbf{F}(y) dy, \quad (17)$$

$$T\mathbf{E} := \mathbf{E}_\tau|_{\Gamma}. \quad (18)$$

Lemma 1. *Operator (18) $T : H \rightarrow W$ is bounded.*

The proof follows from estimate (6.52) in [5, p. 208].

Lemma 2. *Operator (16) $L : W \rightarrow H$ is bounded.*

Proof. We transform the operator L to the form (see [17, p. 93], [5, p. 205])

$$L\mathbf{u} = \text{grad} \int_{\Gamma} \Phi(x, y) \text{div } \mathbf{u}(y) ds(y) + k_0^2 \int_{\Gamma} \Phi(x, y) \mathbf{u}(y) ds(y), \quad (19)$$

where div is again understood as the surface divergence. It is known [17, p. 93], [5, p. 209] that the operator $Kv := \int_{\Gamma} \Phi(x, y) v(y) ds(y)$ is bounded when acting from $H^{-1/2}(\Gamma)$ to $H_{\text{loc}}^1(\mathbf{R}^3)$; therefore, $\text{grad} \int_{\Gamma} \Phi(x, y) \text{div } \mathbf{u}(y) ds(y) \in L_{\text{loc}}^2(\mathbf{R}^3)$ and $\text{curl} \int_{\Gamma} \Phi(x, y) \mathbf{u}(y) ds(y) \in L_{\text{loc}}^2(\mathbf{R}^3)$ for $\mathbf{u} \in W$. Obviously, $\int_{\Gamma} \Phi(x, y) \mathbf{u}(y) ds(y) \in L_{\text{loc}}^2(\mathbf{R}^3)$. In addition, $\text{curl grad div} \int_{\Gamma} \Phi(x, y) \mathbf{u}(y) ds(y) \equiv 0$. Therefore, $L : W \rightarrow L_{\text{Curl}}^2(\Omega_1) \equiv H$ is bounded and we have an estimate $\|L\mathbf{u}\|_H \leq C \|\mathbf{u}\|_W$ with some constant $C > 0$ and any $\mathbf{u} \in W$. Note that it suffices to prove this estimate for smooth functions (tangent fields), and then extend it to the entire space W by continuity due to the denseness of smooth functions in W .

Lemma 3. *If $\varepsilon \in C^\infty(\bar{\Omega}_1)$ and $\varepsilon(x) > \varepsilon_0$ for $x \in \bar{\Omega}_1$, then the operator $I - A : H \rightarrow H$ is bounded and has a continuous inverse.*

Proof. In Theorem 6 [19, p. 160], it is proved that, under the conditions specified in the theorem, the operator $I - A : L_2(\Omega_1) \rightarrow L_2(\Omega_1)$ is bounded and has a continuous inverse. Consider this operator in the space H .

Let us prove the boundedness of the operator A in H . Obviously, $\text{curl} \int_{\Omega_1} \Phi(x, y)(\varepsilon(x)/\varepsilon_0 - 1)\mathbf{F}(y)dy \in L_2(\Omega_1)$ (and even belongs to $H_{\text{loc}}^1(\mathbf{R}^3)$), if $\mathbf{F} \in L_2(\Omega_1)$ (see [19, p. 144]). Therefore, $\int_{\Omega_1} \Phi(x, y)(\varepsilon(x)/\varepsilon_0 - 1)\mathbf{F}(y)dy \in H$ if $\mathbf{F} \in H$. Next, $\text{grad div} \int_{\Omega_1} \Phi(x, y)(\varepsilon(x)/\varepsilon_0 - 1)\mathbf{F}(y)dy \in L_2(\Omega_1)$ if $\mathbf{F} \in L_2(\Omega_1)$ (since the potential $\int_{\Omega_1} \Phi(x, y)(\varepsilon(x)/\varepsilon_0 - 1)\mathbf{F}(y)dy \in H_{\text{loc}}^2(\mathbf{R}^3)$) [19, p. 144]. But $\text{curl grad div} \int_{\Omega_1} \Phi(x, y)(\varepsilon(x)/\varepsilon_0 - 1)\mathbf{F}(y)dy \equiv 0$; therefore, $\text{grad div} \int_{\Omega_1} \Phi(x, y)(\varepsilon(x)/\varepsilon_0 - 1)\mathbf{F}(y)dy \in H$ if $\mathbf{F} \in H$. Then, by virtue of (17), the operator A is bounded in H .

Let $\mathbf{F} \in L_2(\Omega_1)$ be the unique solution of the equation $(I - A)\mathbf{F} = \mathbf{V}$, $\mathbf{V} \in L_2(\Omega_1)$, which holds by virtue of the continuous invertibility of the operator $I - A : L_2(\Omega_1) \rightarrow L_2(\Omega_1)$. Obviously, for $\mathbf{V} \in H$, such a unique solution exists too. Assume that it is $\mathbf{F}$. We will prove that $\mathbf{F} \in H$. We have $\mathbf{F} - A\mathbf{F} = \mathbf{V}$, or $\mathbf{F} = A\mathbf{F} + \mathbf{V}$. But, as shown above, $A : H \rightarrow H$ is bounded. Therefore, $\mathbf{F} = A\mathbf{F} + \mathbf{V} \in H$. By the Banach inverse mapping theorem [20, p. 225], $I - A : H \rightarrow H$ is continuously invertible.

Theorem 1. *If $\varepsilon \in C^\infty(\bar{\Omega}_1)$ and $\varepsilon(x) > \varepsilon_0$ for $x \in \bar{\Omega}_1$ and $\sigma_g(x) \in C^\infty(\Gamma)$, $|\sigma_g(x)| \leq 1$, for $x \in \Gamma$, then there exists $\sigma_0 > 0$ such that, for $\sigma_A \leq \sigma_0$, the operator of Eq. (14)*

$$I - A + \sigma_A \frac{1}{i\omega\varepsilon_0} L(\sigma_g T) : H \rightarrow H \quad (20)$$

is continuously invertible.

Proof. By Lemmas 1–3, operator (20) is the sum of a continuously invertible operator $I - A$ and a bounded operator $\sigma_A \frac{1}{i\omega\varepsilon_0} L(\sigma_g T)$ (as a composition of a bounded operator $T : H \rightarrow W$, the operator of multiplication by an infinitely differentiable function $\sigma_g(x)$, and a bounded operator $L : W \rightarrow H$) with a small norm, if we take a sufficiently small $\sigma_0 > 0$. Then, by the theorem on the sum of a continuously invertible operator and an operator with a small norm [20, p. 229], we obtain the continuous invertibility of the operator under consideration.

Corollary 1. Under the hypotheses of Theorem 1, a solution of Eq. (14) exists and is unique for any smooth $\mathbf{E}_0(x)$ satisfying (1).

Remark 1. The condition of smallness of the conductivity σ_A is natural for graphene in the infrared and terahertz frequency ranges.

In the case of nonlinearity of graphene, the derivation of the integro-differential equation and the results on the solvability of this equation remain valid. It is possible to propose an iterative procedure for solving the nonlinear equation (14), based on the method of contracting mappings, and prove its convergence.

CONCLUSIONS

In this article, we have considered a vector problem of diffraction of an electromagnetic wave by an inhomogeneous dielectric body with a graphene coating, without taking into account the nonlinearity of graphene. A field-matching problem has been formulated and a vector integro-differential equation on the body has been obtained using the Stratton–Chu formulas. A specific feature of this equation is that it contains both the unknown function itself in the region of inhomogeneity and its trace on the boundary of this region. Such an equation can be called a surface-volume equation.

Continuous invertibility of the operator corresponding to the equation has been proved. The existence and uniqueness of a solution of the integro-differential equation in a suitable Sobolev space has been proved as well.

FUNDING

This work was supported by ongoing institutional funding. No additional grants to carry out or direct this particular research were obtained.

CONFLICT OF INTEREST

The author of this work declares that he has no conflicts of interest.

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Translated by E. Chernokozhin

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