

Solving the Inverse Problem for the Gavril'yak–Negami Model in the Detection of Breast Tumors by Impedance Spectroscopy

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Abstract—The inverse problem of calculating coefficients in the Gavril'yak–Negami model has been considered in order to determine the structure of an inhomogeneous dielectric object on the basis of determining the parameters and analyzing the frequency characteristics of the relative permittivity. A method for calculating coefficients in the Gavril'yak–Negami model using three measurements of the complex permittivity at various frequencies has been applied. A numerical method has been developed to determine the coefficients in the Gavril'yak–Negami model, which is used in dielectric spectroscopy for early detection of tumors in the mammary gland.

Keywords: heterogeneous dielectric structure, Gavril'yak–Negami model, impedance spectroscopy, inverse problem, tumor in the mammary gland

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1. INTRODUCTION

Breast cancer is the most common cancer in women. Early diagnosis of this disease is extremely important because it allows for effective treatment. A rapidly developing method for detecting breast tumors is the electro impedance mammography (EIM) that is a specific application of bioimpedance spectroscopy [1], widely used in the study of various biological structures [2, 3]. The EIM method is based on the difference between the complex electrical resistance of cancer and healthy cells [4]. EIM does not use ionizing radiation, which allows conducting examinations without limiting their number and frequency.

Effective treatment of noncommunicable socially significant diseases (malignant tumors, diabetes mellitus, and diseases characterized by high blood pressure) is impossible without the active participation of the patient in the form of self-control. Self-monitoring of diabetes and hypertension at home has instrumental support: glucose meter and blood-pressure monitor. Breast self-examination in order to identify signs of tumor formation does not yet have such support. The use of EIM during self-examination is hindered by the need to organize mechanical contact between the electrodes and the bulkiness of the device. The most promising method for detecting breast tumors during self-examination is one of the methods of bioimpedance spectroscopy (BIS), i.e., dielectric spectroscopy, which is free of these disadvantages. The relative per-

mittivity of a tumor differs significantly from the relative permittivity of healthy breast tissues [4, 5]. This opens up the possibility of early detection of tumors by measuring monitor and control of the dielectric parameters of the object under study in an informative frequency range of a harmonically varying electromagnetic field. The capacitive primary converter does not require mechanical contact between the electrodes and the skin and can be located on the outer surface of the clothes.

When using the dielectric spectroscopy method, the mammary gland is considered as an object of study with an individual shape and size, consisting of a dielectric material with an inhomogeneous structure, parts of which, including tumors, have different relative permittivity. There are known mathematical models of the frequency response of relative permittivity: the Debye model, the Cole–Cole model, etc. The most general mathematical model promising for use in the study of heterogeneous dielectric structures is the Gavril'yak–Negami equation [6]:

$$\varepsilon = \varepsilon_1 + i\varepsilon_2 = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{[1 + (i\omega\tau_0)^{1-\alpha}]^\beta}, \quad (1)$$

where ε_1 is the dielectric constant; ε_2 is dielectric losses; ε_s is the static dielectric constant; ε_∞ is the high-frequency dielectric constant; α is the real coefficient of spectrum width, $\alpha \in [0; 1]$; β is the real coef-

ficient of spectrum asymmetry, $\beta \in [0; 1]$; ω is the circular frequency of sinusoidal voltage; and τ_0 is the relaxation time constant.

It is proposed to determine the structure of an inhomogeneous dielectric object on the basis of determination of the parameters and an analysis of the frequency characteristics of the relative permittivity by solving the direct and inverse diffraction problems. The immediate task is to determine the frequency characteristics of the dielectric constant ε_1 and the dielectric losses ε_2 , as well as the dependence of dielectric losses ε_2 on dielectric constant ε_1 in the informative frequency range of the sinusoidal electric field acting on the measurement object [7].

The inverse problem is to calculate the values of the parameters of the Gavril'yak–Negami model using the known values of components ε_1 and ε_2 obtained as a result of the measurements of the informative parameters of the dielcometer sensor applied to the measurement object at different frequency values.

Many works, in particular [8, 9], have been devoted to determining the coefficients in the Gavril'yak–Negami model, clarifying their physical meaning, and comparing them with experimental data. In this article, we propose another method for determining these coefficients by measuring the components of the relative permittivity at different frequencies.

2. TRANSFORMATION OF THE GAVRILYAK–NEGAMI EQUATION

Let us transform the Gavril'yak–Negami equation for the convenience of solving the inverse problem.

Let Eq. (1) be fulfilled. We introduce the following parameters:

$$A := \frac{\varepsilon_s - \varepsilon_\infty}{\varepsilon - \varepsilon_\infty}, \quad T_\alpha := \tau_0^{1-\alpha}, \quad q := i\omega$$

where A is a complex quantity, T_α is a real quantity, and q is a purely imaginary quantity.

Then Eq. (1) takes the form

$$1 + (i\omega)^{1-\alpha} T_\alpha = A^{\frac{1}{\beta}},$$

or

$$1 + q^{1-\alpha} T_\alpha = A^{\frac{1}{\beta}},$$

or

$$q^{1-\alpha} = \frac{1}{T_\alpha} \left(A^{\frac{1}{\beta}} - 1 \right). \quad (2)$$

Having taken the logarithm of Eq. (2) considering that the quantities included in Eq. (2) are complex

(that is, the logarithm is considered as a multivalued function of a complex variable), we will have:

$$\text{Ln} q = \frac{1}{1-\alpha} \text{Ln} \left(\frac{1}{T_\alpha} \left(A^{\frac{1}{\beta}} - 1 \right) \right). \quad (3)$$

Let

$$\theta := \text{Ln} q. \quad (4)$$

Then, using Eqs. (3) and (4), we obtain the equation

$$\theta = \frac{1}{1-\alpha} \ln \left(\frac{1}{T_\alpha} \right) + \frac{1}{1-\alpha} \text{Ln} \left(A^{\frac{1}{\beta}} - 1 \right). \quad (5)$$

It is considered here that quantity T_α is real and the logarithm of it is also a real function. We will further simplify Eq. (5) by introducing new variables:

$$\alpha_0 := \frac{1}{1-\alpha}, \quad \beta_0 := \frac{1}{\beta}, \quad \text{and} \quad Q := \alpha_0 \ln \left(\frac{1}{T_\alpha} \right). \quad (6)$$

In the new variables of Eqs. (6), Eq. (5) takes the form

$$\theta = Q + \alpha_0 \text{Ln} (A^{\beta_0} - 1). \quad (7)$$

2. THE INVERSE PROBLEM OF RESTORING PARAMETERS USING RESULTS OF MEASUREMENTS OF THE COMPONENTS OF RELATIVE PERMITTIVITY

The initial data for solving the inverse problem of the Gavril'yak–Negami model are the results of measuring static permittivity ε_s at the minimum voltage frequency, high-frequency permittivity ε_∞ at the maximum voltage frequency, and the results of measuring permittivity and dielectric losses at three intermediate frequencies. The inverse Gavril'yak–Negami problem is solved by the methods of the theory of functions of a complex variable, as well as numerical methods.

Let the measurements be made at frequencies ω_1 , ω_2 , and ω_3 . We denote $A_1 := A(\omega_1)$, $A_2 := A(\omega_2)$, and $A_3 := A(\omega_3)$; and $\theta_1 := \theta(\omega_1)$, $\theta_2 := \theta(\omega_2)$, and $\theta_3 := \theta(\omega_3)$.

Let us construct a system of equations:

$$\begin{cases} \theta_1 = Q + \alpha_0 \text{Ln} (A_1^{\beta_0} - 1) \\ \theta_2 = Q + \alpha_0 \text{Ln} (A_2^{\beta_0} - 1) \\ \theta_3 = Q + \alpha_0 \text{Ln} (A_3^{\beta_0} - 1) \end{cases} \quad (8)$$

Subtracting the first equation in Eqs. (8) from the second and third equations, we have

$$\begin{cases} Q = \theta_1 - \alpha_0 \text{Ln}(A_1^{\beta_0} - 1) \\ \theta_2 - \theta_1 = \alpha_0 \text{Ln} \frac{A_2^{\beta_0} - 1}{A_1^{\beta_0} - 1} \\ \theta_3 - \theta_2 = \alpha_0 \text{Ln} \frac{A_3^{\beta_0} - 1}{A_2^{\beta_0} - 1} \end{cases} \quad (9)$$

Dividing the third equation in Eqs. (9) by the second one, we obtain

$$\frac{\theta_3 - \theta_2}{\theta_2 - \theta_1} = \frac{\text{Ln} \frac{A_3^{\beta_0} - 1}{A_2^{\beta_0} - 1}}{\text{Ln} \frac{A_2^{\beta_0} - 1}{A_1^{\beta_0} - 1}}, \quad (10)$$

Note that

$$B := \frac{\theta_3 - \theta_2}{\theta_2 - \theta_1} = \frac{\ln \frac{\omega_3}{\omega_2}}{\ln \frac{\omega_2}{\omega_1}}, \quad (11)$$

is a real quantity.

Equation (10), taking Eq. (11) into account, can be written as

$$F(\beta_0) - B = 0, \quad (12)$$

where the function

$$F(\beta_0) := \frac{\text{Ln} \frac{A_3^{\beta_0} - 1}{A_2^{\beta_0} - 1}}{\text{Ln} \frac{A_2^{\beta_0} - 1}{A_1^{\beta_0} - 1}}, \quad (13)$$

is a complex-valued function of a real variable.

Next, we solve Eq. (12) on a set $\beta_0 \in [1; \beta_{\max}]$ for a sufficiently large $\beta_{\max} > 1$.

After determining β_0 as a solution of Eq. (12), we find α_0 sequentially from the formula (from the second equation in Eqs. (9)):

$$\alpha_0^{-1} = \frac{1}{\theta_2 - \theta_1} \text{Ln} \frac{A_2^{\beta_0} - 1}{A_1^{\beta_0} - 1}, \quad (14)$$

then, from Q , from the first equation in Eqs. (9), and from Eq. (6), we obtain

$$T_\alpha = \frac{1}{\exp(Q/\alpha_0)}, \quad (15)$$

Next, from Eqs. (6) and (15), we find the parameters in the Gavril'yak–Negami model that need to be found in the inverse problem:

$$\tau_0 = T_\alpha^{\alpha_0}, \quad (16)$$

$$\alpha = 1 - \frac{1}{\alpha_0}, \quad (17)$$

$$\beta = \frac{1}{\beta_0}. \quad (18)$$

Equations (16)–(18) give a solution to the inverse problem.

3. NUMERICAL METHOD FOR SOLVING THE INVERSE PROBLEM

In the proposed scheme for solving the inverse problem, only Eq. (12), with Eq. (13) and the constant Eq. (11), is numerically solved. The solution consists in determining the points of change of sign of the real and imaginary parts of the function $F(\beta_0) - B$, that is, points β_0^* such that

$$\text{Re}(F(\beta_0^*) - B) = 0, \quad (19)$$

and

$$\text{Im}F(\beta_0^*) = 0. \quad (20)$$

A uniform grid with a sufficiently small step h is selected on the segment $[1; \beta_{\max}]$, and the step is preliminarily estimated by plotting functions from the left-hand sides of Eq. (19) and Eq. (20). The values of these functions are determined at the nodes of the grid. If the sign of both functions changes on some partial segment of the grid (the functions have different signs at different ends of the partial segment), we consider the middle of this partial segment to be the approximate root of Eq. (12) and, accordingly, the approximate value for β_0^* .

4. CALCULATION RESULTS

The results of calculations of the coefficients of the Gavril'yak–Negami model using the proposed method are presented below and compared with the experimental data published in [10].

Figure 1 shows a graph of the real and imaginary parts of the complex permittivity calculated by Eq. (1) depending on $\log(\omega)$, where $\epsilon_1 = \epsilon'$ and $\epsilon_2 = -\epsilon''$. The experimental data at the three frequencies $\omega_1 = 10$ Hz, $\omega_2 = 100$ Hz, and $\omega_3 = 1000$ Hz presented in Table 1 were selected in accordance with [6]. Thus, the relative error for the real parts $\epsilon(\omega_i)$, $i = 1, 2, 3$ is 0.05%. The largest relative error is the error for the imaginary part $\epsilon(\omega_1)$, which is approximately 1%. The remaining imaginary parts have an error of about 0.05%. The calculation results are presented in Table 2.

The above comparisons show that the method proposed in the article provides a satisfactory solution to the inverse problem. Of course, this is true only if the

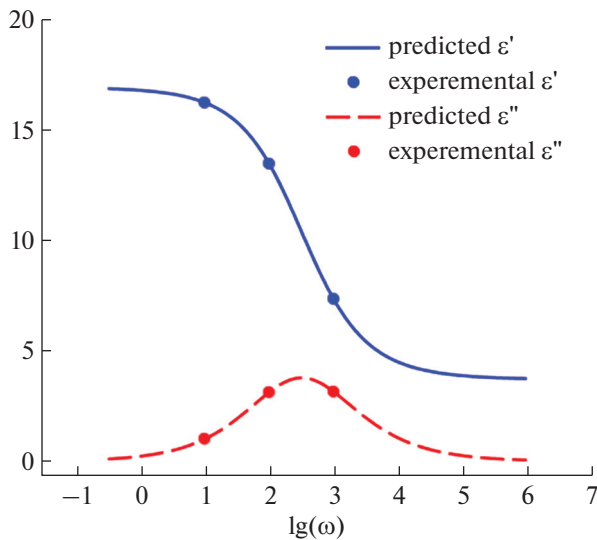


Fig. 1. Frequency dependences of real and imaginary parts of relative complex permittivity $\varepsilon = \varepsilon' - i\varepsilon''$ at $\alpha = 0.33$, $\beta = 0.96$, $\tau_0 = 3.3 \times 10^{-3}$ s, $\varepsilon_s = 16.93$, and $\varepsilon_\infty = 3.71$.

Table 1. Exact and experimental values of relative complex permittivity

	Exact values	Experimental values
$\varepsilon(\omega_1)$	$16.237 - 1.011i$	$16.229 - 1.022i$
$\varepsilon(\omega_2)$	$13.475 - 3.126i$	$13.469 - 3.128i$
$\varepsilon(\omega_3)$	$7.359 - 3.159i$	$7.355 - 3.158i$

Table 2. Exact values of parameters and those obtained by the method described in this work

	Exact values	Experimental values
β	0.96	0.98
α	0.33	0.335
τ_0	3.3×10^{-3}	$3.134604384 \times 10^{-3}$

real frequency dependences are very well approximated using the Gavriljak–Negami model.

It should be noted that, when calculating the coefficients, because of unavoidable rounding, complex values (with a small imaginary part) can be obtained. In this case, it is necessary to take the real part of the calculated parameter.

5. CONCLUSIONS

The article proposes a method for determining the parameters of the Gavriljak–Negami model on the basis of the known values of components ε_1 and ε_2 obtained as a result of the measurements of the infor-

mative parameters of the dielcometer sensor applied to the measurement object at three different frequency values.

The method involves the numerical solution of a complex equation on a segment, more precisely, the search for zeros of a complex-valued function of a real variable on a certain segment. To do this, the method of changing the sign is used simultaneously for the real and imaginary parts of the function. A computational algorithm is presented, and test calculations are performed. A comparison with known experimental data is carried out.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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